2.2 TRACTIVE EFFORT AND RESISTANCE

$$F_f + F_r = ma + R_a + R_{rlf} + R_{rlr} + R_g$$

$$F = ma + R_a + R_{rl} + R_g$$

2.3 AERODYNAMIC RESISTANCE

$$R_a = \frac{\rho}{2} C_D A_f V^2 \qquad \text{hp}_{R_a} = \rho\, C_D A_f V^3$$

2.4 ROLLING RESISTANCE

$$R_{rl} = f_{rl}\, W \qquad \text{hp}_{R_{rl}} = f_{rl} WV$$

$$f_{rl} = 0.01\left(1 + \frac{V}{147}\right)$$

2.5 GRADE RESISTANCE

$$R_g = W \sin\theta_g$$

$$R_g \cong W \tan\theta_g$$

$$R_g = WG$$

2.6 AVAILABLE TRACTIVE EFFORT

2.6.1 Maximum Tractive Effort

$$F_{max} = \frac{\mu W \left(l_r + f_{rl} h \right) / L}{1 + \mu h / L}$$

2.6.2 Engine-Generated Tractive Effort

$$F_e = \frac{M_e \varepsilon_0 \eta_d}{r} \qquad \text{hp}_e = \frac{2\pi M_e n_e}{550}$$

$$V = \frac{2\pi r n_e \left(1 - i \right)}{\varepsilon_0}$$

2.7 VEHICLE ACCELERATION

$$F - \sum R = \gamma_m m a \qquad \gamma_m = 1.04 + 0.0025 \varepsilon_0^2$$

$$a = \frac{F - \sum R}{\gamma_m m}$$

2.9 PRINCIPLES OF BRAKING

$$F_{bf\ max} = \mu W_f = \frac{\mu W}{L}\left[l_r + h(\mu + f_{rl})\right]$$

$$F_{br\ max} = \mu W_r = \frac{\mu W}{L}\left[l_f - h(\mu + f_{rl})\right]$$

2.9.2 Braking Force Ratio and Efficiency

$$BFR_{f/r\ max} = \frac{l_r + h(\mu + f_{rl})}{l_f - h(\mu + f_{rl})}$$

$$PBF_f = 100 - \frac{100}{1 + BFR_{f/r\ max}}$$

$$PBF_r = \frac{100}{1 + BFR_{f/r\ max}}$$

$$\eta_b = \frac{g_{\max}}{\mu}$$

2.9.4 Theoretical Stopping Distance

$$K_a = \frac{\rho}{2} C_D A_f \qquad R_a = K_a V^2$$

$$S = \frac{\gamma_b W}{2gK_a} \ln \left[\frac{\mu W + K_a V_1^2 + f_{rl} W \pm W \sin\theta_g}{\mu W + K_a V_2^2 + f_{rl} W \pm W \sin\theta_g} \right]$$

$$S = \frac{\gamma_b W}{2gK_a} \ln \left[1 + \frac{K_a V_1^2}{\mu W + f_{rl} W \pm W \sin\theta_g} \right]$$

$$S = \frac{\gamma_b W}{2gK_a} \ln \left[1 + \frac{K_a V_1^2}{\eta_b \mu W + f_{rl} W \pm W \sin\theta_g} \right]$$

$$S = \frac{\gamma_b \left(V_1^2 - V_2^2 \right)}{2g \left(\eta_b \mu + f_{rl} \pm \sin\theta_g \right)}$$

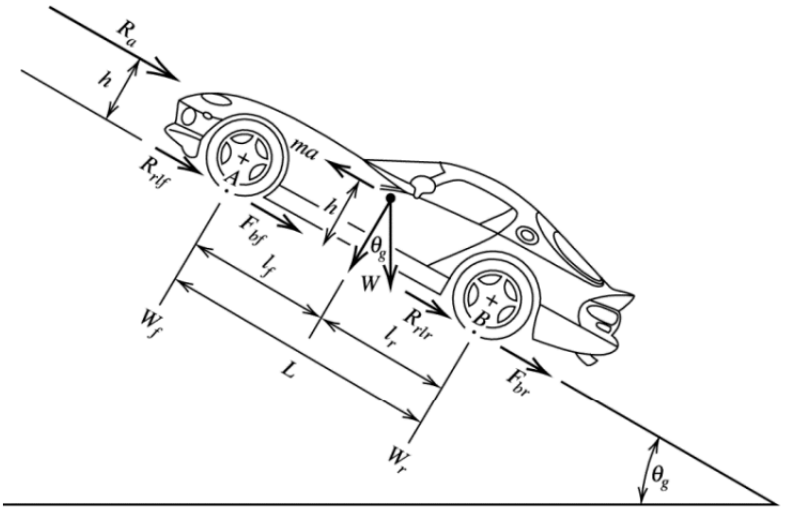
2.9.5 Practical Stopping Distance

$$d = \frac{V_1^2}{2g \left(\left(\frac{a}{g} \right) \pm G \right)} \qquad d_r = V_1 \times t_r$$

$$d_s = d + d_r$$

$$S = \frac{V_1^2}{2g \left(\eta_b \mu \pm G \right)}$$

NOMENCLATURE



a	acceleration (deceleration if negative)
A_f	frontal area of vehicle
$BFR_{f/r\ max}$	brake force ratio (front over rear) for maximum braking force
C_D	coefficient of aerodynamic drag
d	practical stopping distance
d_a	distance to accelerate
d_r	distance traveled during driver perception/reaction
d_s	total stopping distance (vehicle braking distance plus perception/reaction distance)
F	total available tractive effort
F_b	total braking force
F_{bf}	front-axle braking force
$F_{bf\ max}$	maximum front-axle braking force
F_{br}	rear-axle braking force
$F_{br\ max}$	maximum rear-axle braking force
F_e	engine-generated tractive effort
F_f	available tractive effort at the front axle
F_{max}	maximum tractive effort
F_r	available tractive effort at the rear axle
f_{rl}	coefficient of rolling resistance
G	roadway grade in ft/ft (percent grade divided by 100; $G = 0.05$ is a 5% grade)
g	gravitational constant (32.2 ft/s^2)
g_{max}	maximum deceleration achieved before wheel lockup
h	height of vehicle's center of gravity above the roadway surface
hp_e	engine-generated power, measured in horsepower
i	drive axle slippage
K_a	elements of aerodynamic resistance that are not a function of speed
L	vehicle wheelbase
l_f	distance from vehicle's center of gravity to front axle
l_r	distance from vehicle's center of gravity to rear axle
M_e	engine torque
m	mass
n_e	engine speed in crankshaft revolutions per second
PBF_f	optimal percent of braking force on the front axle
PBF_r	optimal percent of braking force on the rear axle
R_a	aerodynamic resistance
R_g	grade resistance
R_{rl}	rolling resistance
r	radius of vehicle drive wheels
S	minimum theoretical stopping distance
t_r	driver perception/reaction time
V	vehicle speed
W	total vehicle weight
W_f	vehicle weight acting normal to the roadway surface on the front axle
W_r	vehicle weight acting normal to the roadway surface on the rear axle
γ_b	braking mass factor
γ_m	acceleration mass factor
ε_0	gear reduction ratio
η_b	braking efficiency
η_d	drivetrain efficiency
θ_g	angle of grade
μ	coefficient of road adhesion
ρ	air density